Oblivious RAM

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Architectual Approach to Software Protection

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- Technologically possible: accessable memory but protected processor
- Today: making interction between processor and memory useless for learning program

Outline

1. What Kind of Computer Are We Going To Construct?

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2. Basic Solutions

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Computer Model

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- Internal memory of Processor $= c \log |\text{Memory}|$
- Interaction: fetch(adress), store(adress, value)
- Processor has access to random oracle
- Computation starts with a program and an input in Memory
- One step: fetch one cell - update value and Processor memory - store

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Weaker requirement:

For all programs of size m working in time t
order of fetch/store adresses has the same distribution

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Simulation 1:

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Cost of simulation: tm time, m memory

Square Root Solution (1)

We need to protect:

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Idea:

Divide computation in epochs of $\sqrt{m}$ steps each

On each original step make one fetch to the Main Part and scan through all the Shelter

Square Root Solution (2)

Simulation 2:

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   - For each process(i) scan through the shelter. If i-th element is not founded, fetch it from $\pi(i)$, otherwise fetch next dummy cell

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Cost of simulation: $t\sqrt{m}$ time, $m + 2\sqrt{m}$ memory

Buffer Solution (1): Oblivious Hash Table

Memory of initial program: $(a_1, v_1), \dots, (a_m, v_m)$

- Take a hash function $h : [1..m] \to [1..m]$
- Prepare $m \times \log m$ table
- Put (a_i, v_i) to random free cell in $h(a_i)$-th column
- Home problem 4: Prove that the chance of overflow is less than $1/m$

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Restricted problem: assume that every cell accessed only once

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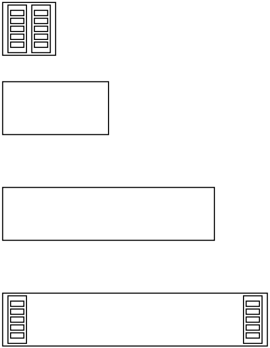
Cost of simulation: $t \log m$ time, $m \log m$ memory

Outline

Data Structure

- k-Buffer = table $2^k \times k$
- Hierarchial Buffer Structure = 1-buffer,...,$\log t$-buffer
- Initial position: input in last buffer, all others are empty



Hierarchial Simulation

Simulation of processing cell i:

1. Scan through 1-buffer

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Periodic Rehashing

Refreshing the data structure:

1. Every 2^{j-1} steps unify j-th and $j-1$-th buffers
2. Delete doubles
3. Using new hash function put all data to $j-th$ level

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Invariant: For every moment of time for every l buffers from 1 to l all together contain at most 2^{l-1} elements

Discussion

Comments on final solution:

- Cost: $O(t \cdot (\log t)^3)$ time, $O(m \cdot (\log m)^2)$ memory

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Comments on final solution:

- Cost: $O(t \cdot (\log t)^3)$ time, $O(m \cdot (\log m)^2)$ memory
- Omitted details: realization of oblivious hashing and random oracle
- Tamper-proofing extension

Home Problem 4

Prove that the chance of overflow in hash table construction is less than $1/m$

Summary

Main points:

- Theoretical model for hardware-based code protection: open memory/protected CPU

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- Central problem: simulation of any program with any input by the same access pattern

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- Theoretical model for hardware-based code protection: open memory/protected CPU
- Central problem: simulation of any program with any input by the same access pattern
- Current result: $O(t \cdot (\log t)^3)$ time, $O(m \cdot (\log m)^2)$ memory simulation

Reading List

O. Goldreich, R. Ostrovsky
Software protection and simulation on oblivious RAM, 1996.
http://www.wisdom.weizmann.ac.il/~oded/PS/soft.ps.

Thanks for attention. Questions?